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Discussion

Author's Closure: Discussion on "Exact expansions of arbitrary tensor functions F(A) and their derivatives"

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I would like to thank Dr. Dui for his comments and interest in my work. Judged from his comments, I feel that some aspects of the paper may not have been appreciated. For this reason, I would like to clarify all the issues that Dr. Dui raised.

1. The paper deals with tensor power series. The title of the paper may be somewhat misleading, but I hope that the scope has been made clear in the abstract as well as in the opening sentence of the introduction. The work was initially inspired by Kusnezov's 1995 paper on exponential map; while working on it I realized his method can be generalized to any power series. Isotropic functions $\mathbf{F}(\mathbf{A}) = \alpha_0 + \alpha_1\mathbf{A} + \alpha_2\mathbf{A}^2$ where α_i's are functions of invariants of $\mathbf{A}$ are not the target of the paper. In fact, these functions may not be derivable from scalar potentials.
2. In my opinion, an intrinsic merit of the generating function $G(\mathbf{A})$ lies in that it reveals the differential root of tensor power series. As Dr. Dui correctly pointed out, it is not necessary to find the explicit form $G(\mathbf{A})$ (or $g(x)$) if one only wants to obtain an invariant representation for $\mathbf{F}(\mathbf{A})$. For this reason, it is not necessary to find the closed form of the integral $\int x^{-1}\sin x\,dx$ as appeared in his example. However, if $g(x)$ can be easily found, we have the luxury of deriving alternative representations. This point has been demonstrated in the paper with the example of the inverse function of a tensor.
3. Regarding loss smoothness, I disagree with Dr. Hui in saying that that there is a serious problem with this approach. It is known that if a function $\varphi(\lambda_i)$ originally defined in terms of the eigenvalues of $\mathbf{A}$ is represented in terms of the principal invariants, loss of differentiability will occur because the derivatives of eigenvalues relative to the principal invariants are not defined at repeated eigenvalues. However, if the function $\varphi(\lambda_i)$ starts sufficiently smooth, the ensuing invariants form would be smooth, although at a

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lower order. Indeed, it can be seen from Eqs. (75)–(77) that, if $f(x)$ is C^5, the coefficients in (75)–(77) are at least C^0. The function $f(x)$ considered in the paper is sufficiently smooth as it is defined by the power series. Therefore, smoothness should not be an issue here.

4. I agree with Dr. Dui in that, for *diagonalizable* tensors there are shortcuts in deriving the representation for $\mathrm{d}\mathbf{F}(\mathbf{A})/\mathrm{d}\mathbf{A}$. A common approach is to express the rate of $\mathbf{F}(\mathbf{A})$ in terms of isotropic function of $\mathbf{A}$ and $\dot{\mathbf{A}}$ using representation theorems, and find the coefficients by matching values at selected points. However, it is important to notice that, the results so obtained *do not apply to non-diagonalizable tensors*. Take Dr. Dui's representation (5) as an example, consider
$$[\mathbf{A}] = \begin{bmatrix} \lambda & 1 & 0 \\ 0 & \lambda & 1 \\ 0 & 0 & \lambda \end{bmatrix}.$$
One can readily check that $\widehat{\mathbf{A}}$, as defined in the equation immediately below (5), is
$$[\widehat{\mathbf{A}}] \equiv [(I_1^3 - 4I_2)I + 2I_1A - 3A^2] = \begin{bmatrix} 0 & 0 & -3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$
Therefore $\det \widehat{\mathbf{A}} = 0$. One immediately sees that (5) is singular since $\det \widehat{\mathbf{A}}$ appears in a denominator in (5).

5. I want to thank Dr. Dui for bringing up the work by Balendran and Nemat-Nasser, which has been inadvertently overlooked when the paper was written. The two papers dealt with the same problem from different approaches, and they apparently ended with the same representations.